Shrinkage in Space

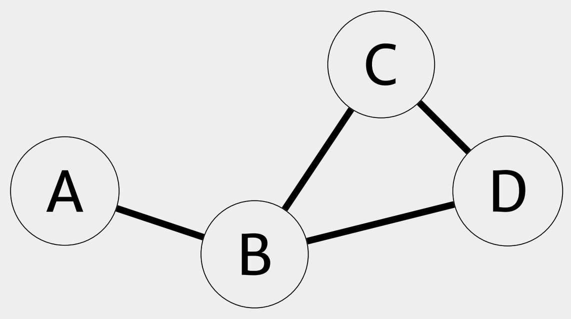
Priors for Spatial Econometrics

Nikolas Kuschnig (nkuschnig@wu.ac.at)
Vienna University of Economics and Business

I. Introduction

Connectivity between units and **spillovers** between them are central elements of many applied research questions. Economic markets, for instance, are a collection of links between participants, which is reflected in macroeconomic dynamics [1], supply chains [2], and labour markets [8].

Econometric methods for network data focus on the *structure of connectivities*, or *spillover effects*. Despite recent progress, there is a lack of integrated models [4].



I develop a **fully Bayesian approach** to the standard network model, explicitly modelling network structures. For this, I review *uncertainty* and *prior information* in the setting, and propose a *shrinkage prior* for the overall connectivity strength. This facilitates comprehensive models that model individual connectivity strength and structural parameters, such as the distance-decay or the number of neighbours.

II. Status quo

The standard 'linear-in-means' model [7] is given by

$$\mathbf{y} = \lambda \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}, \qquad (1)$$

where the non-negative matrix $\mathbf{W} \in \mathbb{R}^{n \times n}$ imposes a structure and is **assumed to be known**. The spatial parameters are

- ➔ λ, an *autoregressive* term, and
- ➔ $\boldsymbol{\theta}$, capturing local *interference*.

A useful formulation uses a latent $\mathbf{z}$ and a filter $\mathbf{S} = (\mathbf{I} - \lambda \mathbf{W})$,

$$\mathbf{z} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon},$$
$$\mathbf{y} = \mathbf{S}(\lambda|\mathbf{W})^{-1}\mathbf{z}.$$

We will focus on the filter $\mathbf{S}(\lambda|\mathbf{W})$ and its elements. The support of the only free parameter λ is such that $\mathbf{S}$ is invertible and possibly stationary—usually $\lambda \in (-1, 1)$ after scaling $\mathbf{W}$. Priors are usually flat, generalised by $\lambda_s = (\lambda + 1)/2 \sim \text{Beta}$ [6].

III. Issues

The standard approach has two severe shortcomings—we

1. **discount** the **connectivity structure** by fixing $\mathbf{W}$,
2. **cannot express prior information** on λ effectively.

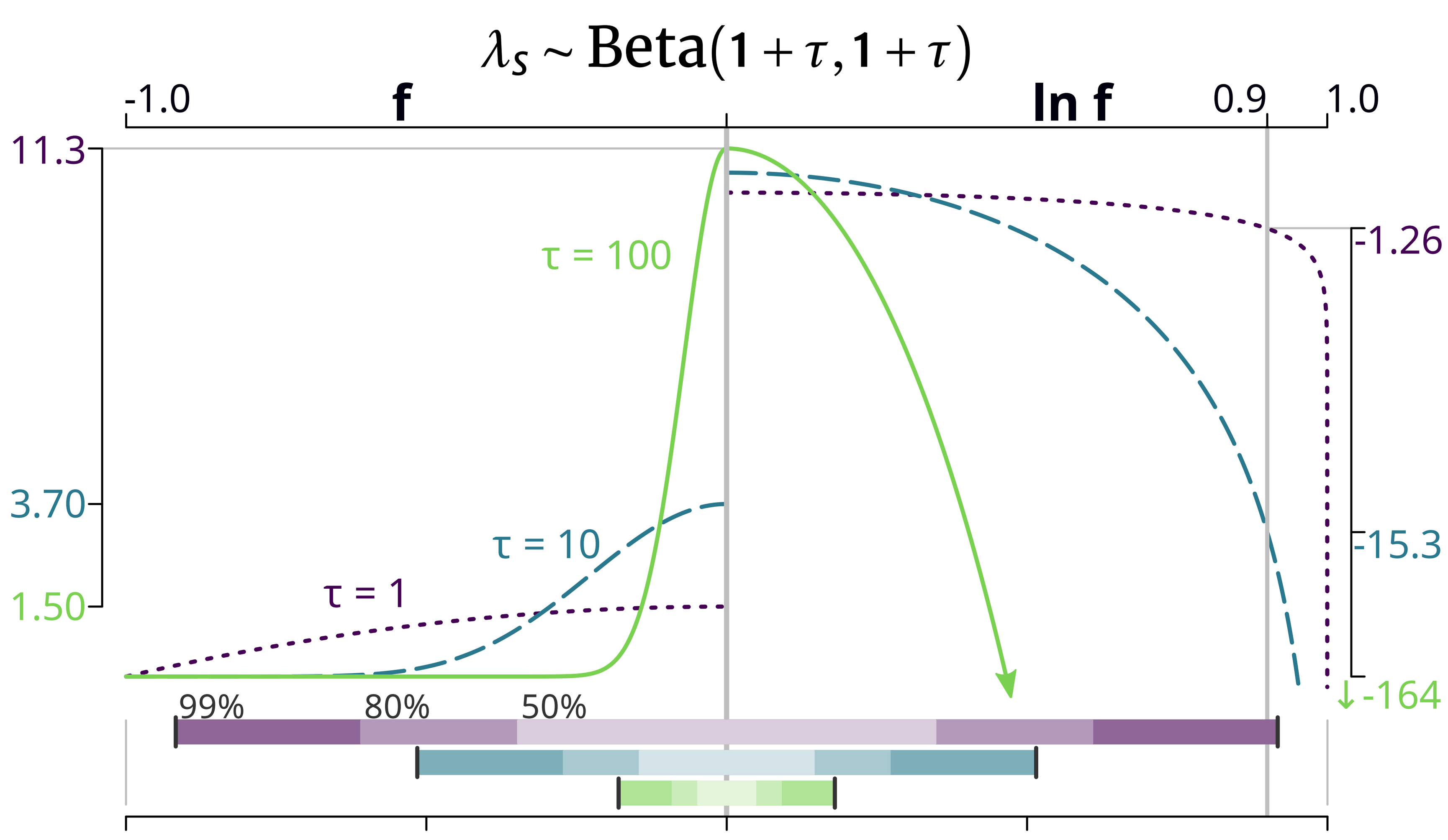


Figure: Density (left), log-density (right), and 99% / 80% / 50% credible intervals (bottom) of λ_s, scaled such that $\lambda \in (-1, 1)$, with increasing prior information $\tau \in \{1, 10, 100\}$.

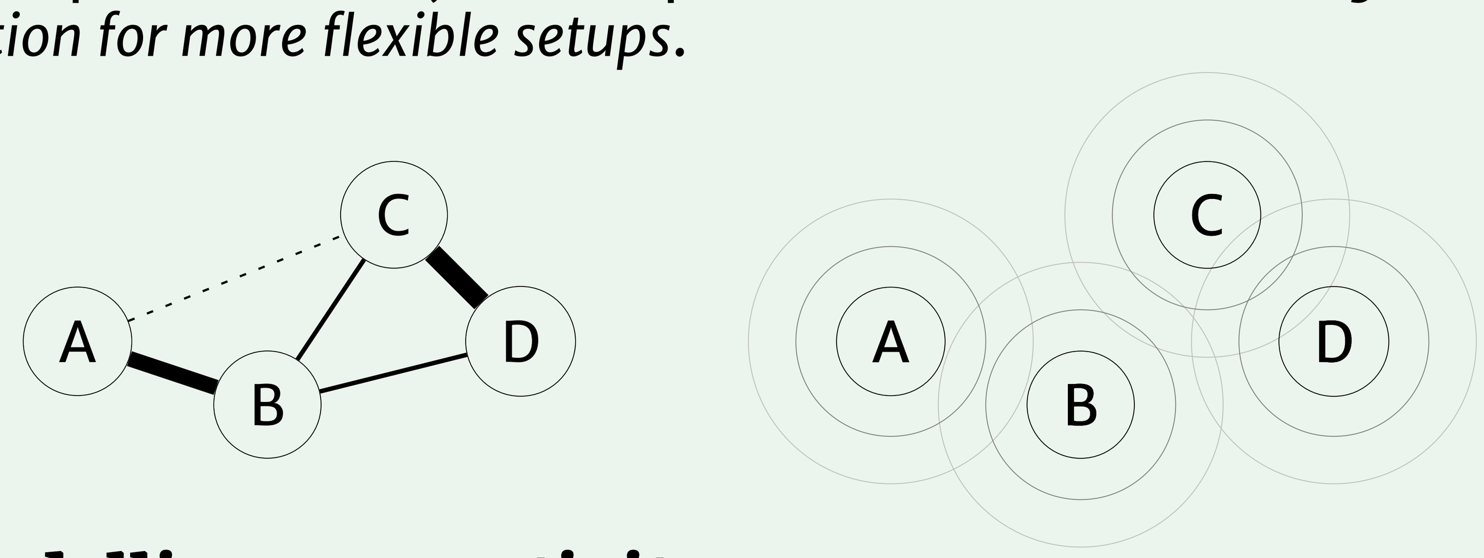
IV. Solutions

Shrinkage. A hierarchical prior allows greater flexibility

$$p(\lambda_s|\tau) \sim \text{Beta}(1 + \tau, 1 + \tau),$$
$$p(\tau) \sim \text{Gamma}(\alpha, \beta),$$

- ➔ where β induces a sharp **spike at zero**,
- ➔ and α guarantees **support throughout**.

This prior (a) arguably better reflects *prior information* [3], (b) is parsimonious, and (c) provides much-needed *regularisation for more flexible setups*.



Modelling connectivity. $\mathbf{W}$ is usually constructed as a *function of the locations*, e.g. based on inverse-distance decay, and scaled, e.g. using the spectral radius $\rho(\mathbf{W})$. For instance, $\mathbf{W} = \text{diag}(\boldsymbol{\xi})\Psi$, where $\psi_{ij} = d_{ij}^{-\delta}$, and $\xi_i = \rho(\Psi)^{-1}$.

A useful extension of Equation 1 considers $\mathbf{S}(\lambda, \delta|\mathbf{D})$, with

$$p(\delta) \sim \text{Gamma}^{-1}(\delta_a, \delta_b).$$

Estimation. We can get estimates of our extended model with Markov chain Monte Carlo methods. We draw from

- ➔ $p(\tau|\lambda_s, \cdot)$, using a Gibbs-step with a *rejection sampler*,
- ➔ $p(\delta|\cdot)$, using a Metropolis–Hastings sampler.

Computing the Jacobian $|\mathbf{S}|$ can be prohibitive, and approximations are used commonly [6]. I propose an efficient alternative for higher dimensions using *Gaussian process approximation*.

V. Application

I demonstrate by revisiting *agricultural spillovers* on Amazon deforestation [5], and focus on the partial effect of croplands

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}_{crops}} = \mathbf{S}(\lambda, \delta)^{-1}(\mathbf{I}\beta_{crops}).$$

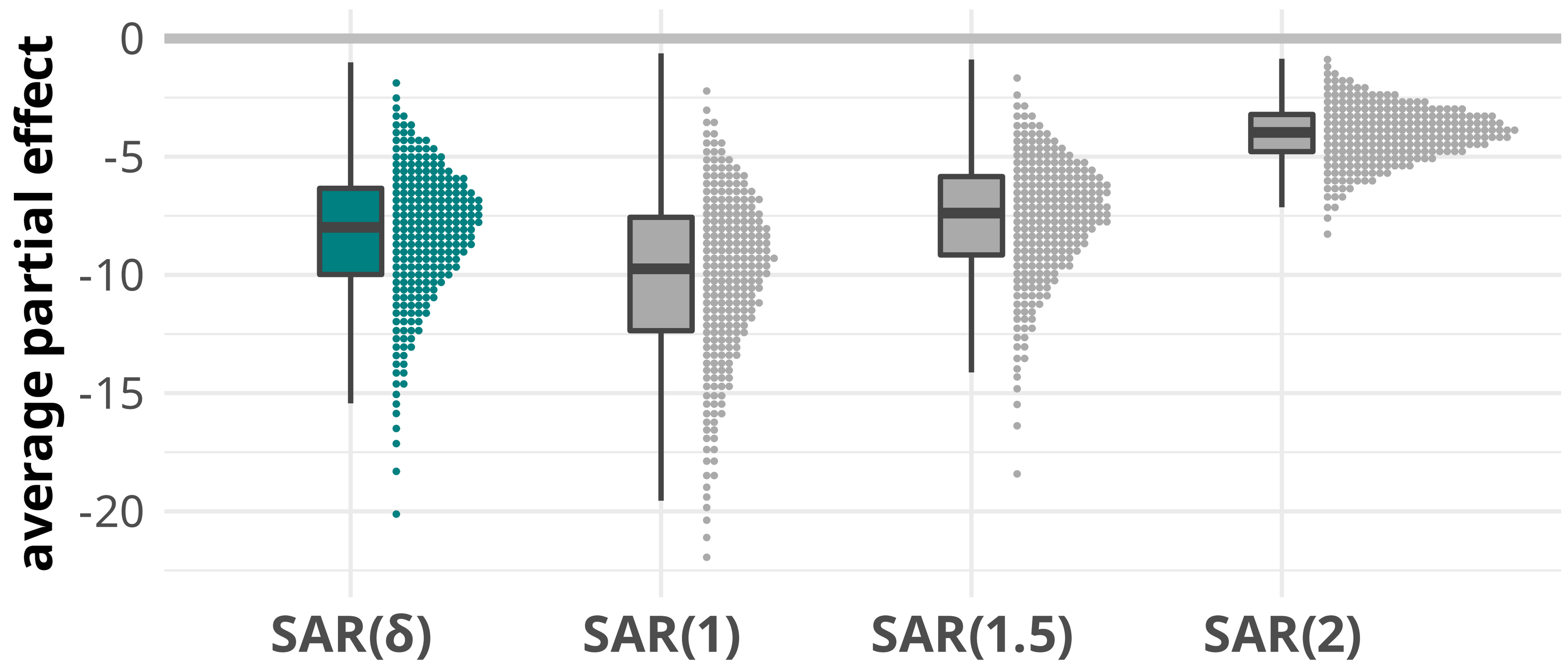


Figure: Posterior partial effects using spatial models with $\mathbf{S}(\lambda, \delta)$, and $\mathbf{S}(\lambda)$ with fixed δ.

Contact

Find my work online at kuschnig.eu (QR code); contact me via mail at nkuschnig@wu.ac.at or messenger pigeon at twitter.com/__nkuschnig.

[1] Acemoglu, D., Carvalho, V.M., Ozdaglar, A. et al, 2012. The network origins of aggregate fluctuations. *Econometrica*, 80(5). doi:10.3982/ECTA9623.

[2] Atalay, E., Hortaçsu, A., Roberts, J. et al, 2011. Network structure of production. *PNAS*, 108(13). doi:10.1073/pnas.1015564108.

[3] Bramoullé, Y., Djebbari, H. and Fortin, B., 2020. Peer effects in networks: a survey. *ARE*, 12(1). doi:10.1146/annurev-economics-020320-033926.

[4] Graham, B.S., 2020. Network data. In *HoE*. doi:10.1016/bs.hoe.2020.05.001.

[5] Kuschnig, N., Crespo Cuaresma, J., Krisztin, T. et al, 2021. Spillover effects in agriculture drive deforestation in Mato Grosso, Brazil. *SciRep*, 11(1). doi:10.1038/s41598-021-00861-y.

[6] LeSage, J. and Pace, R.K., 2009. *Introduction to spatial econometrics*. doi:10.1201/9781420064254.

[7] Manski, C.F., 1993. Identification of endogenous social effects: the reflection problem. *REStud*, 60 (3). doi:10.2307/2298123.

[8] Munshi, K., 2003. Networks in the modern economy: Mexican migrants in the U.S. labor market. *QJE*, 118(2). doi:10.1162/003355303321675455.